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EFFECT OF DIELECTRIC-LAYER ON THE STRESS FIELD OF MICRO CANTILEVER BEAMS AT THE ONSET OF PULL-IN INSTABILITY

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ABSTRACT

In this paper, stress distribution of micro cantilever beams in the presence of a dielectric-layer is studied using an analytic method. The Modified Adomian Decomposition Method (MADM) is applied to obtain a semi-analytical solution for a distributed parameter model of the micro cantilever beam. The important parameters for designing and manufacturing micro-actuators such as shear force, bending moment and stress distribution along the cantilevers are computed for different values of the dielectric-layer parameter. The results of MADM are compared with the numerical results, and they found in good agreement. It is found that increase of the dielectric-layer parameter increases the dimensionless pull-in voltage, tip deflection, internal stress and bending moment of the micro cantilever actuators at the onset of pull-in instability.

Keywords: Stress resultants, Micro cantilever, Pull-in, Dielectric layer.

1. INTRODUCTION

Many micro-devices consist of beams or plate electrodes suspended above a ground plane. The suspended beams or plates have many applications such as pressure sensors, accelerometers, rate gyroscopes, electrical switches, optical switches, chemical sensors, adaptive optical devices, electrostatic actuators, valves and pumps and resonators [1-3].

A typical MEMS actuator is a micro-beam electrode suspended above a conductive substrate. Applying a voltage difference between the micro cantilever beam and the ground plane causes the micro-beam to buckle toward to the substrate due to electro static forces. At a critical voltage, (pull-in voltage) the micro-beam electrode pulls-in onto the substrate plane and instability occurs [4]. Different models such as lumped model, 1D distributed model, planar model and 3D simulation enhanced with the finite-element method (FEM) have been used to study the deflection and pull-in parameters of various beam structures [4].

The lumped model of MEMS actuators with one degree of freedom results in easy calculations [5-7], but it fails to capture details of the internal filed stress in the MEMS. At the other end, 3D-models can evaluate the deflection and internal stress filed of the micro-cantilever actuators, but they need expensive calculations, thus they are time consuming [8]. Furthermore, numerical simulation of MEMS near the pull-in instability may result in divergence of the solution. The two-dimensional distributed parameter models at an intermediate level of complexity provide useful results with a reasonable computational effort [9].

Legtenberg *et al*. [10] proposed a cantilever beam model to obtain the characteristics of large-displacement actuators. Mullen *et al*. [11] applied the finite-element method to simulate buckling behavior of micro fabricated beams. Chan *et al*. [12] considered the effects of electro static fringing field and finite beam thickness on the pull-in voltage and capacitance-voltage of a two-dimensional model of micro beams. A mixed-regime approach is introduced by Li and Aturu [13] to combine linear and nonlinear theories of beams to handle large buckling of MEMS under substantial applied voltages. Moreover, the nanocantilever beams immersed in a liquid electrolyte are theoretically investigated by Noghrehabadi *et al*. [14]. The modified couple stress theory also is utilized to interpret the size effect which appears in micro/nanoscale structures. They found that the presence of electrolyte affects the critical pull-in parameters of micro/nano actuators.

Recently, power series methods are employed to obtain analytical solutions for pull-in instability of micro and nano actuators. Kuang and Chen [15] applied the modified Adomian decomposition method to investigate the nonlinear pull-in behavior of different types of micro-actuators. Ghalambaz *et al*. [16] utilized a symbolic power series method based on Taylor polynomial expansions to obtain free standing length, electrostatic pull-in instability and stress resultants of cantilevers. In a different work, they applied a monotone iterative method to obtained deflection of cantilevers in 2D distributed parameter model [17]. In the both works electrostatic forces, fringing field effect and intermo-

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lecular forces have been taken into account. The Adomian decomposition method (ADM) has been used by Soroush *et al*. [18] and Koochi *et al*. [19] to obtain deflection and pull-in parameters of cantilever actuators subjected to intermolecular and electrical forces. It has been shown that the Modified Adomian Decomposition Method (MADM) is a powerful and convenient method which can effectively solve micro and nano mechanics problems [20-24].

However, the pull-in instability of beam actuators has been studied in the large number of papers, but only few studies considered the effect of dielectric-layers on the pull-in instability characteristics and stress resultants at the onset of pull-in instability in micro cantilever actuators. Rollier *et al*. [25] investigated the effect of dielectric-layer on the pull-in stability of MEMS both experimentally and analytically. They used lamped parameter model in liquids for their analytical solution and showed that the pull-in effect can be shifted beyond the one-third of the gap and can even be suppressed. They demonstrated that insulating layers of the actuator plates play a major role in this phenomenon. Gorthi *et al*. [9] used CoventorWare™ software to simulate the behavior of electrostatic actuators before and beyond the pull-in instability in the presence of a dielectric-layer. They have studied transition forms of these actuators, and classified all possible types of transitions based on the dielectric-layer parameters. Recently, the influence of capillary force on the deflection and pull-in instability of fixed-fixed electrostatic micro actuator beams in the presence of a dielectric layer is investigated by Yazdanpanahi *et al*. [26]. They show that there are distinct values of the dielectric parameter in which the variation of the dimensionless capillary parameter does not affect the values of maximum deflection, internal stress and bending moment of the micro actuator at the onset of pull-in instability. These special values of dielectric parameter are introduced as the Balance Dielectric Layer (BDL) because of their unique properties.

Manufacturing reliable micro-actuators requires crucial knowledge about the pull-in performance and internal stress field of the micro structures. Knowledge about stress distribution over the length of the micro switch and its buckling behaviour is essential for design process of these MEMS devices to avoid failures in action [27-29].

In the present paper, the modified Adomian decomposition method is implemented to analyze the effect of dielectric-layer on the internal stress field of cantilever micro-actuators. The solution obtained by MADM is compared with numerical data.

2. MATHEMATICAL MODEL

Figure 1 shows the schematic view of a micro-cantilever beam, suspended above a fixed substrate with a bended end at $x = 0$ and external voltage difference, V, between the beam and substrate. There is a dielectric-layer with thickness of t_1 above the substrate layer, and

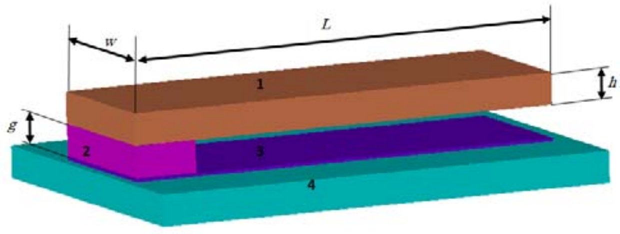

Fig. 1 Schematic representation of distributed model of micro cantilever-beam with a dielectric-layer

the length of the cantilever beam is L. The rectangular cross-section of the beam assumed uniform with thickness h and width w. The gap between the dielectric-layer and cantilever is g that remains constant by the increase of the dielectric thickness.

It is assumed that the constitutive material of the micro-cantilever is linear elastic, and the finite kinematic effects are negligible when $L > 10$ g [30]. Thus, only long beams are considered in this study. This simplification is acceptable for most cases [18,19,31]. For the small values of g/w the fringing field effect is negligible. Hence, wide micro cantilevers solely are considered in this study. Here, the electrostatic force per unit length of the micro-cantilever, electrical force, can be defined as [9,25],

$$f_{elec} = \frac{\varepsilon_0 \, \varepsilon w V^2}{2\left(\dfrac{t_1 \varepsilon}{\varepsilon_1} + g - Y\right)^2}, \quad (1)$$

where $\varepsilon_0 = 8.854 \times 10^{-12} \text{c}^2/\text{Nm}^2$ is the permittivity of vacuum, and ε is the relative permittivity of the medium. t_1 is the thickness and ε_1 is the relative permittivity of the insulating layer. V is the external voltage difference between substrate and cantilever. Considering only the static elastic small deflection of the micro cantilever beam and employing the virtual work principle, the appropriate approximation of the beam deflection can easily be found in the absence of the non-conservative forces [18]. As there are not deflection and rotation at the fixed end and due to the absence of the bending moment and shear force at the free end of the beam, the deflection of the micro cantilever beam can be defined as the following boundary-value differential equation,

$$E_{eff} I \frac{d^4 Y}{dX^4} = f_{elec}, \quad (2a)$$

where the geometrical boundary conditions at the fixed end are,

$$Y(0) = Y'(0) = 0, \quad (2b)$$

and natural boundary conditions at the free end are,

$$Y''(l) = Y'''(l) = 0, \quad (2c)$$

where Y is the deflection of the beam, X is the position along the beam measured from the clamped end, prime denotes differentiation with respect to X.

In the case of $w > 5h$ the effective modulus E_{eff} can be approximated by the plate modulus $E/(1-v^2)$; otherwise E_{eff} is the Young's modulus E [32]. I is the moment of inertia of the beam cross section [33]. By substituting Eq. (1) into Eq. (2) and introducing the non- dimensional variables as,

$$\beta = \frac{\varepsilon_0 \varepsilon w L^4 V^2}{2 g^3 E_{eff} I}, \quad K = \frac{t_1 \varepsilon}{\varepsilon_1 g}, \quad x = \frac{X}{L}, \quad u = \frac{Y}{g}, \tag{3}$$

The mathematical model can be parameterized in the non-dimensional form for convenience as,

$$\frac{d^4 u}{dx^4} = \frac{\beta}{\left[K + 1 - u(x)\right]^2}, \tag{4a}$$

subject to the following conditions,

$$u(0) = u'(0) = 0, \quad \text{at} \quad x = 0, \tag{4b}$$

$$u''(1) = u'''(1) = 0, \quad \text{at} \quad x = 1, \tag{4c}$$

where β is the non-dimensional applied voltage, and K is the dielectric-layer parameter.

3. MODIFIED ADOMIAN DECOMPOSITION METHOD

The modified Adomian decomposition was established as a very effective, simple and convenient method to solve nonlinear initial and boundary value problems. The main idea of this method is explained in Wazwaz [20]. Here, Eq. (4) by using the substitution of $y(x) = 1 - u(x)$ can be rewritten as,

$$\frac{d^4 y}{dx^4} = N\left(y(x)\right), \tag{5a}$$

subject to,

$$y(0) = 1, \quad y'(0) = 0, \quad \text{at} \quad x = 0, \tag{5b}$$

$$y''(1) = y'''(1) = 0, \quad \text{at} \quad x = 1, \tag{5c}$$

where

$$N\left(y(x)\right) = -\frac{\beta}{\left[K + y(x)\right]^2}, \tag{5d}$$

Equation (5a) can be represented as the following form,

$$L^{(4)}\left[y(x)\right] = N\left(y(x)\right), \tag{6}$$

where $L^{(4)}$ is fourth order differential operator which is defined as,

$$L^{(4)} = \frac{d^{(4)}}{dx^{(4)}}, \tag{7}$$

The inverse operator $L^{-(4)}$ can be define as a 4-fold integral operator as [15],

$$L^{-(4)} = \int_0^x \int_0^x \int_0^x \int_0^x (.) dx dx dx dx, \tag{8}$$

By using the introduced operator and applying the modified Adomian decomposition method to the Eq. (5a) [20], the dependent variable, $y(x)$, can be written as [34],

$$y(x) = \sum_{n=0}^{\infty} y_n(x) = C_0 + C_1 x + \frac{1}{2} C_2 x^2 + \frac{1}{6} C_3 x^3 - L^{-(4)}\left[N\left(y(x)\right)\right], \tag{9a}$$

where the constants of C_0 to C_3 are,

$$C_0 = y(0), \quad C_1 = y'(0), \tag{9b}$$

$$C_2 = y''(0), \quad C_3 = y'''(0), \tag{9c}$$

where $N(y(x))$ is the nonlinear function at the right side of the Eq. (5a), and it can be approximated by series of Adomian polynomials,

$$N\left(y(x)\right) = \sum_{n=0}^{\infty} A_n(x), \tag{10}$$

Implementing the boundary condition at $x = 0$ and using the above equation, Eq. (9a) can be summarized as,

$$y(x) = \sum_{n=0}^{\infty} y_n(x) = 1 + \frac{1}{2} C_2 x^2 + \frac{1}{6} C_3 x^3 - L^{-(4)}\left[\sum_{n=0}^{\infty} A_n(x)\right], \tag{11}$$

where C_2 and C_3 can be evaluated later by the solution of the algebraic equations come from the boundary conditions at $x = 1$ (*i.e.* Eq. (9c)). According to the modified Adomian decomposition method from [20], the recursive relations of Eq. (11) can be demonstrated as follows,

$$y_0(x) = 1, \tag{12a}$$

$$y_1(x) = \frac{1}{2} C_2 x^2 + \frac{1}{6} C_3 x^3 - L^{-(4)}\left[A_0(x)\right], \tag{12b}$$

$$y_k(x) = -L^{-(4)}\left[A_k(x)\right], \tag{12c}$$

Here, the Adomian polynomial A_n can be determined by the following convenient equations [20,21],

$$A_n = \sum_{v=1}^{n} C(v, n) H_v(y_0) \quad (n > 0), \tag{13a}$$

$$C(v, n) = \sum_{p_i} \prod_{i=1}^{v} \frac{y_{pi}^{ki}}{k!}, \quad \left(\sum_{i=1}^{v} k_i\, p_i = n,\ 0 \le i \le n,\ 1 \le p_i \le n - v + 1\right), \tag{13b}$$

$$H_v(y_0) = d^v / dy_0^v \left[N(y_0)\right], \tag{13c}$$

where k_i is the number of repetition in y_{pi} and the values of p_i are selected from the above range by combination without repetition. Expanding Eq. (13a) yields,

$$A_0 = N(y_0) = \frac{1}{(K+y_0)^2}, \tag{14a}$$

$$A_1 = C(1,1)\,H_1(y_0) = y_1 H_1(y_0) = -\frac{2y_1}{(K+y_0)^3}, \tag{14b}$$

$$\begin{aligned} A_2 &= C(1,2)H_1(y_0) + C(2,2)H_2(y_0) \\ &= y_2 H_1(y_0) + \frac{1}{2!} y_1^2 H_2(y_0) \\ &= -\frac{2y_2}{(K+y_0)^3} + \frac{3y_1^2}{(K+y_0)^4}, \end{aligned} \tag{14c}$$

$$\begin{aligned} A_3 &= C(1,3)H_1(y_0) + C(2,3)H_2(y_0) + C(3,3)H_3(y_0) \\ &= y_3 H_1(y_0) + y_1 y_2 H_2(y_0) + \frac{1}{3!} y_1^3 H_3(y_0) \\ &= -\frac{2y_3}{(K+y_0)^3} + \frac{6y_1 y_2}{(K+y_0)^4} - \frac{4y_1^3}{(K+y_0)^5}, \end{aligned} \tag{14d}$$

The Eq. (12) by using Eq. (14) becomes,

$$y_1(x) = \frac{1}{2}C_2 x^2 + \frac{1}{6}C_3 x^3 - \frac{\beta x^4}{24(K+1)^2}, \tag{15a}$$

$$y_2(x) = \frac{\beta C_2 x^6}{360(K+1)^3} + \frac{\beta C_3 x^7}{2520(K+1)^3} - \frac{\beta^2 x^8}{20160(K+1)^5}, \tag{15-b}$$

$$\begin{aligned} y_3(x) = &-\frac{\beta C_2^2}{2240(K+1)^4}x^8 - \frac{\beta C_2 C_3}{6048(K+1)^4}x^9 \\ &+ \frac{47\beta^2 C_2}{1814400(K+1)^6}x^{10} \\ &- \frac{\beta C_3^2}{60480(K+1)^4}x^{10} + \frac{107\beta^2 C_3}{19958400(K+1)^6}x^{11} \\ &- \frac{107\beta^3}{239500800(K+1)^8}x^{12}, \end{aligned} \tag{15c}$$

Therefore, using modified Adomian technique, the polynomial solution of Eq. (5a) is obtained for four terms, which can be summarized as,

$$y(x) = y_0(x) + y_1(x) + y_2(x) + y_3(x) + \ldots, \tag{16a}$$

$$\begin{aligned} y(x) = 1 &+ \frac{C_2}{2}x^2 + \frac{C_3}{6}x^3 - \frac{\beta}{24(K+1)^2}x^4 + \frac{\beta C_2}{360(K+1)^3}x^6 \\ &+ \frac{\beta C_3}{2520(K+1)^3}x^7 \\ &- \left(\frac{\beta^2}{20160(K+1)^5} + \frac{\beta C_2^2}{2240(K+1)^4}\right)x^8 \\ &- \frac{\beta C_2 C_3}{6048(K+1)^4}x^9 \\ &+ \left(\frac{47\beta^2 C_2}{1814400(K+1)^6} - \frac{\beta C_3^2}{60480(K+1)^4}\right)x^{10} \\ &+ \frac{107\beta^2 C_3}{19958400(K+1)^6}x^{11} - \frac{107\beta^3}{239500800(K+1)^8}x^{12}, \end{aligned} \tag{16b}$$

The undetermined coefficients C_2 and C_3 are equivalent to the second and third derivatives of the beam deflection with respect to x at $x = 0$, accordingly. The coefficients of C_2 and C_3 will be evaluated later by using the boundary conditions at the free end of the cantilever beam (*i.e.* Eqs. (5b) and (5c)). Finally, using the modified Adomian decomposition method, the polynomial solution of Eq. (5) is obtained and can be summarized in Eq. (16b). In order to verify the accuracy and convergence of the obtained power series, the deflection of a typical micro actuator is evaluated using different series size of MADM, and it is compared with the numerical results. The numerical results are obtained using the Runge - Kutta- Fehlberg method [35,36].

The variation of the cantilever tip deflection (u_{tip}) of a typical cantilever beam for different sizes of the MADM series is shown in Table 1 when $K = 1.184$ and $\beta = 17$. The results of this table reveal that the higher accuracy can be achieved by evaluating more terms of the modified Adomian series. The relative error shown in Table 1 has been computed from the following equation,

$$Error = \left|\frac{u_{tip,MADM} - u_{tip,Num}}{u_{tip,Num}}\right|, \tag{17}$$

where $u_{tip,MADM}$ and $u_{tip,Num}$ are the micro cantilever tip deflection evaluated from the analytical and the numerical method, respectively, and the Error represents the relative error.

The results of Table 1 show the analytical solution converges to the numerical solution as the number of the selected terms increase. The evaluated values of C_2 and C_3 for the micro-cantilever of Table 1 using eight terms (*i.e.* x^{28}) of the modified Adomian decomposition method are 2.983 and 5.145, respectively. The results of Table 1 represent that the eight terms of the modified Adomian series have the global error less than 0.5% which shows that they are in good agreement with the numerical results. Hence, eight terms of modified Adomian series are selected in the following text for convenience.

Table 1 The variation of the tip deflection of a micro cantilever beam (u_{tip}) evaluated with different selected terms of MADM for K = 1.184 and β = 17

Tip deflection	5 terms (x^{16})	6 terms (x^{20})	7 terms (x^{24})	8 terms (x^{28})	9 terms (x^{32})
MADM	0.74664	0.77471	0.78858	0.79479	0.79737
Numerical	0.79904				
Error %	6.56	3.05	1.31	0.53	0.21

4. INSTABILITY STUDY

In order to study the instability of the micro-actuators in the presence of a dielectric-layer, Eq. (5) is solved numerically, and the results are compared with equation Eq. (16b). For any given β and K, the cantilever tip deflection at the onset of pull-in can be obtained from Eq. (16b) by setting $du(1)/d\beta \rightarrow \infty$. No physical solution exists for $u(x)$ by increasing β beyond β_{PI}.

It is found that for K_{CR} = 1.184 and β_{PI} = 17.6, evaluated using MADM, the cantilever's tip reaches to the dielectric-layer due to deflection. These values cause the maximum deflection and consequently the maximum bending moment and maximum shear stress in the cantilever. The numerical method computed these values as K_{CR} = 1.258, β = 19.3.

5. STRESS RESULTANTS

The stress distribution in any arbitrary cross-section of the micro-beam can be evaluated from the variation of stress resultants (bending moment and shear force) [32]. In order to determine internal stress resultants, the dimensionless internal shear force (F) and bending moment (M) of the beam can be defined as [18],

$$F = \frac{\bar{F}L^3}{E_{eff}Ig}\,, \tag{18a}$$

$$M = \frac{\bar{M}L^2}{E_{eff}Ig}\,, \tag{18b}$$

where $\bar{F}$ is the internal shear force and $\bar{M}$ is the bending moment at the arbitrary cross-section of the beam. Figures 2 and 3 show distribution of F and M along the beam at the onset of instability for different values of the dielectric-layer parameter K, respectively. These figures reveal that the bending moment decreases with the increase of x. The maximum value of bending moment and shear stress occur at x = 0 while the minimum value of them occurs at x = 1. These figures reveal that the increase of the dielectric-layer parameter increases the bending moment and shear stress.

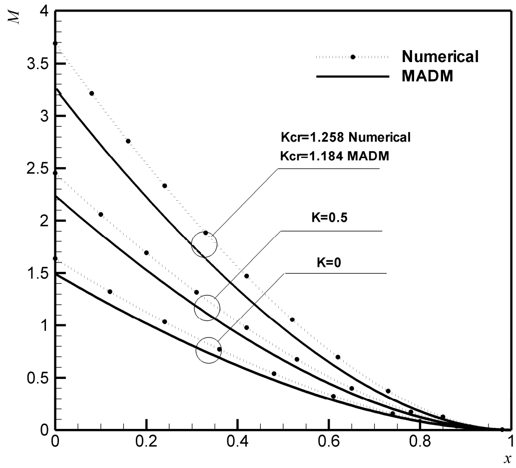

Fig. 2 Distribution of momentum (M) along micro beams at the onset of instability for different values of dielectric-layer parameter

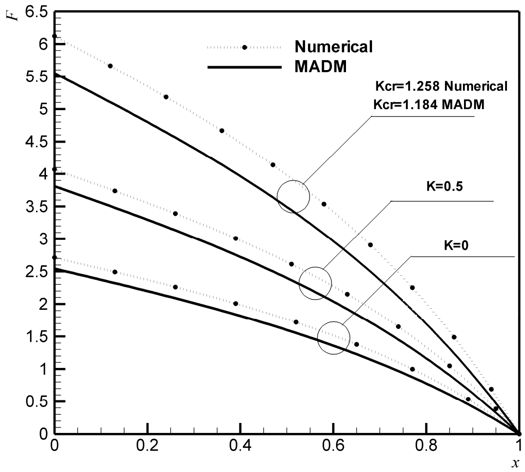

Fig. 3 Distribution of shear force (F) along micro beams at the onset of instability for different values of dielectric-layer parameter

The variation of $F_{PI,\max}$ and $M_{PI,\max}$ of micro cantilevers (or $C_{3,PI}$, $C_{2,PI}$) as a function of β_{PI} and K for different values of the dielectric-layer parameter is presented in Figs. 4 and 5, respectively.

For cantilevers with $h << L$ the magnitude of $\sigma_{PI,\max}$ is much larger than $\tau_{PI,\max}$. Therefore, $\sigma_{PI,\max}$ is usually the primary consideration in design process. For these micro cantilevers, the following relations easily can be derived,

$$\sigma_{PI,\max} = F_{PI,\max}\frac{ghE_{eff}}{2L^2}\,, \tag{19a}$$

$$\tau_{PI,\max} = M_{PI,\max}\frac{gh^2E_{eff}}{8L^3}\,, \tag{19b}$$

where $\sigma_{PI,\max}$ is the maximum normal stress, and $\tau_{PI,\max}$ is the maximum shear stress in the cantilever beam. In the case of K_{CR} = 1.184, β = 17.6 the values of $F_{PI,\max}$ and $M_{PI,\max}$ are 3.681 and 6.071, respectively. Table 2

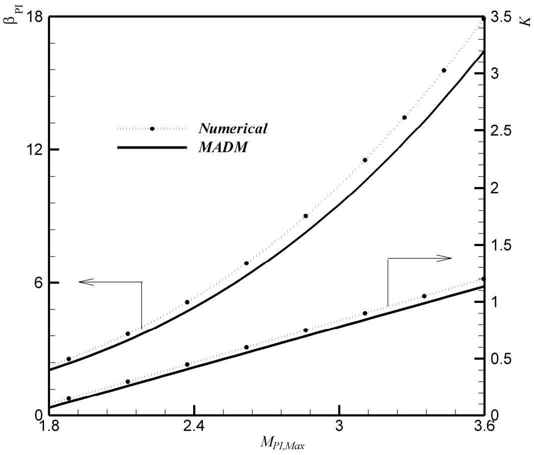

Fig. 4 Relationship of β_{PI} and K with the maximum resultant moments at pull-in instability of micro actuators

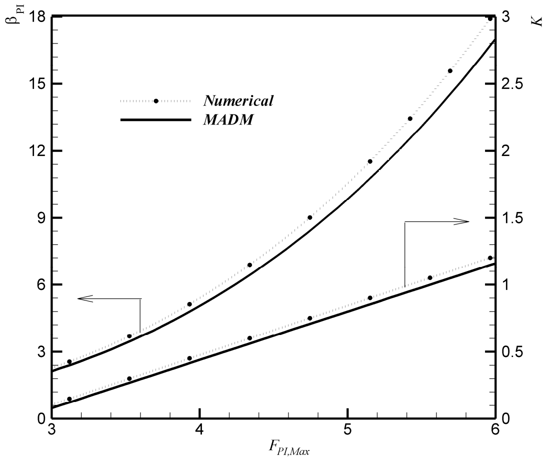

Fig. 5 Relationship of β_{PI} and K with the maximum shear force at pull-in instability of micro actuators

Table 2 Relation of maximum dimensionless internal shear force ($F_{PI,\max}$), bending moment ($M_{PI,\max}$), pull-in voltage (β_{PI}) and tip deflection of micro-cantilevers for variation of dielectric-layer parameter obtained by MAD method

K	β_{PI}	U_{PI}	$M_{PI,\max}$	$F_{PI,\max}$
0	1.691	0.4570	1.6834	2.7772
0.1	2.251	0.5027	1.8517	3.0549
0.2	2.923	0.5487	2.0210	3.3338
0.3	3.716	0.5943	2.1892	3.6113
0.4	4.641	0.6404	2.3586	3.8904
0.5	5.709	0.6863	2.5276	4.1690
0.6	6.928	0.7321	2.6964	4.4472
0.7	8.310	0.7779	2.8652	4.7254
0.8	9.865	0.8238	3.0343	5.0041
0.9	11.602	0.8693	3.2019	5.2810
1	13.532	0.9157	3.3726	5.5615
1.1	15.665	0.9611	3.5397	5.8378
1.184	17.621	0.9995	3.6811	6.0711

shows the relation of dimensionless pull-in voltage (β), maximum dimensionless internal shear force (F), maximum dimensionless bending moment (M) and tip deflection of micro cantilevers at the onset of pull-in instability for variation of the dielectric-layer parameter obtained using MAD method. Results of this table reveal that the increase of the dielectric-layer parameter increases the non-dimensional pull-in voltage, tip deflection, bending moment and shear stress at the onset of instability.

6. CONCLUSIONS

A distributed beam model and the modified Adomian decomposition were utilized to analyze micro cantilever electrostatic actuators with an intermediate dielectric-layer. The Pull-in parameters and deflection of micro-cantilevers in the presence of dielectric-layer were computed using modified Adomian decomposition method as well as the numerical method. The results of the present study demonstrate that the increase of the dielectric-layer parameter increases the non-dimensional pull-in voltage, tip deflection, internal stress and bending moment of micro-cantilever actuators at the onset of instability. The analytical solution was compared with the numerical solution. It is found that the results of MADM with eight terms (*i.e.* x^{20}) are in good agreement with the numerical results. Therefore, the modified Adomian decomposition method could easily evaluate the maximum stress induced in the cantilevers at the onset of instability. The results of present paper can be useful in designing the micro-actuators considering the effect of a dielectric- layer parameter. Furthermore, the proposed closed- form MAD solution makes the parametric study possible.

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NOMENCLATURES

$\bar{F}$	internal shear force at arbitrary cross-section of the beam
$\bar{M}$	internal bending moment at arbitrary cross-section of the beam
A_n	Adomian polynomial
C_2	dimensionless internal bending moment
C_3	dimensionless internal shear force
E	Young's modulus
E_{eff}	effective Young's modulus
F	dimensionless internal shear force
F_{elec}	electrical force
g	initial gap between the upper electrode and the ground plane

h	cantilever beam thickness
I	moment of inertia of the beam cross section
K	dielectric-layer parameter
L	length of cantilever beam
M	dimensionless internal bending moment
N	nonlinear function
t_1	thickness of insulating layer
nvu	non-dimensional deflection
V	applied voltage between the upper electrode and the ground plane
w	width of cantilever beam
X	position along the beam measured from the clamped end
x	non-dimensional position along the beam
Y	distance between cantilever beam and dielectric-layer
y	non-dimensional distance between cantilever beam and dielectric-layer
β	non-dimensional voltage
ε	relative permittivity of the medium
ε_0	permittivity of vacuum
ε_1	relative permittivity of the dielectric layer
σ	dimensionless normal stress at the pull-in instability
τ	dimensionless shear stress at the pull-in instability

Subscripts

CR	Critical
$MADM$	modified Adomian decomposition method
max	Maximum
Num	Numerical
PI	pull-in
tip	value at the free end of the cantilever

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