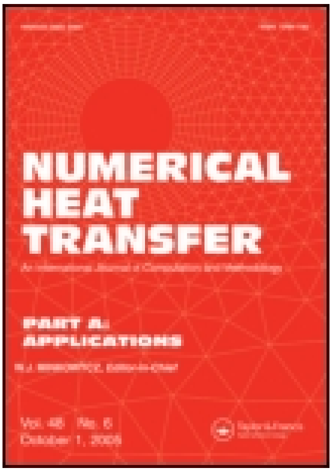

Numerical Heat Transfer, Part A: Applications: An International Journal of Computation and Methodology
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/unht20

NATURAL-CONVECTIVE HEAT TRANSFER IN A SQUARE CAVITY WITH TIME-VARYING SIDE-WALL TEMPERATURE

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Published online: 01 Sep 2006.

Numerical Heat Transfer, Part A, 47: 621–631, 2005

ISSN: 1040-7782 print/1521-0634 online
DOI: 10.1080/10407780590896853

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The natural-convective heat transfer in an inclined square enclosure is studied numerically. The top and bottom horizontal walls are adiabatic, and the right side wall is maintained at a constant temperature T_0. The temperature of the opposing vertical wall varies by sine law with time about a mean value T_0. The system of Navier–Stokes equations in Boussinesq approximation is solved numerically by the control-volume method with SIMPLER algorithm. The enclosure is filled with air ($Pr = 1$) and results are obtained in the range of inclination angle $0^\circ \leq \alpha \leq 90^\circ$ for two values of Grashof number (2×10^5 and 3×10^5). It can be noted that there is a nonzero time-averaged heat flux through the enclosure at $\alpha \neq 0^\circ$. The dependencies of time-averaged heat flux on oscillation frequency and inclination angle are depicted. It is found that the maximal heat transfer corresponds to the values of inclination angle $\alpha = 54^\circ$ and dimensionless frequency $f = 20\pi$ for both Grashof numbers studied (2×10^5 and 3×10^5).

INTRODUCTION

Unsteady natural-convective heat transfer in enclosures under periodic variation of boundary conditions has attracted significant interest for the last two decades. For instance, the case of a square cavity with time-dependent wall temperature has been reported in [1] and [2] (this latter article also contains a rather detailed review of studies handling similar problems). The increasing interest is attributable to the relevance of such transient processes in many technological applications. Typical examples are heating and cooling of buildings, solar energy utilization, thermal energy storage, and cooling of electronic equipment, to name a few.

The stability characteristics and fluid motion behavior in this problem have been studied in [3], where the influence of frequency and amplitude of boundary temperature oscillation on the convective flow pattern and critical Rayleigh number have been discussed for an inclined rectangular enclosure of width-to-height aspect ratio 4:1.

Received 24 July 2004; accepted 17 September 2004.

A preliminary version of this article was presented at CHT-04: An ICHMT International Symposium on Advances in Computational Heat Transfer, April 2004, G. de Vahl Davis and E. Leonardi (eds.), CD-ROM Proceedings, ISBN 1-5670-174-2, Begell House, New York, 2004.

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NOMENCLATURE

c	heat capacity	u, v	velocity components in x and y directions
f	dimensionless oscillation frequency $(\omega\rho_0 L^2/\mu)$	U, V	dimensionless velocity components $(u/v^*$ and $v/v^*)$
g	gravitational acceleration	v^*	reference velocity $(\mu/\rho_0 L)$
Gr	Grashof number $(g\beta T_1 L^3 \rho_0^2/\mu^2)$	x, y	Cartesian coordinates
L	length of cavity side	X, Y	dimensionless Cartesian coordinates $(x/L$ and $y/L)$
Nu_{av}	time-averaged heat transfer through the cavity in a period	α	inclination angle
Nu_r	averaged heat flux through right wall	β	coefficient of thermal expansion
P	pressure	θ	dimensionless temperature $[(T - T_0)/T_1]$,
P^*	reference pressure $(\rho_0 v^{*2})$	λ	thermal conductivity coefficient
$\tilde{P}$	dimensionless pressure $\{[P + \rho_0 g(x \sin\alpha + y\cos\alpha)]/P^*\}$	μ	coefficient of dynamic viscosity
Pr	Prandtl number $(c\mu/\lambda)$	ρ_0	density
t	time	τ^*	reference time $(\rho_0 L^2/\mu)$
T	temperature, K	τ	dimensionless time (t/τ^*)
T_0, T_1	constant temperatures, K	ω	oscillation frequency

All the previous studies assumed that the time-mean temperature at one of the walls is higher (or lower) than the steady temperature of the other wall. In the present article the boundary conditions are different, and time-averaged temperature on the wall is equal to the corresponding constant temperature of the opposite wall. This leads to the zero time-averaged heat flux for the case of vertical heat exchanging walls, but it is different from zero in an inclined cavity.

The influence of inclination angle and oscillation frequency on heat transfer through the square enclosure is studied for two values of Grashof number.

FORMULATION OF THE PROBLEM

The heat transfer in an inclined square enclosure filled with fluid is considered (Figure 1). The top and bottom horizontal walls are adiabatic, and the right side wall is maintained at a constant temperature T_0. The temperature of the opposing vertical wall varies by sine law with time about a mean value T_0.

The flow is assumed to be laminar and two-dimensional. The fluid is incompressible and its thermophysical properties are treated as constant everywhere, except for the density in the body force term in the momentum equations.

The system of equations in Boussinesq approximation can be written as follows:

$$\rho_0 \frac{\partial u}{\partial t} + \rho_0 u \frac{\partial u}{\partial x} + \rho_0 v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \mu\,\Delta u - \rho_0 g[1 - \beta(T - T_0)]\ \sin\alpha \tag{1}$$

$$\rho_0 \frac{\partial v}{\partial t} + \rho_0 u \frac{\partial v}{\partial x} + \rho_0 v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \mu\,\Delta v - \rho_0 g[1 - \beta(T - T_0)]\ \cos\alpha \tag{2}$$

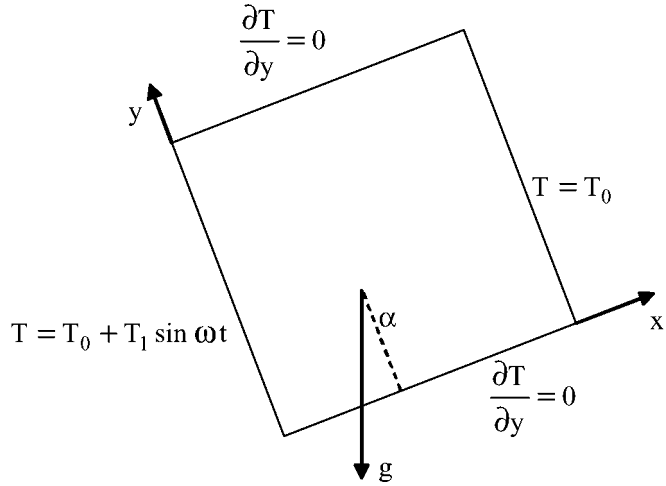

Figure 1. Enclosure geometry and temperature boundary conditions.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$\rho_0 c\frac{\partial T}{\partial t}+\rho_0 cu\frac{\partial T}{\partial x}+\rho_0 cv\frac{\partial T}{\partial y}=\lambda\,\Delta T \tag{4}$$

The boundary conditions are

$$x=0: \qquad T=T_0+T_1\sin\omega t \tag{5}$$

$$x=L: \qquad T=T_0 \tag{6}$$

$$y=0: \qquad \frac{\partial T}{\partial y}=0 \qquad u=v=0 \tag{7}$$

$$y=L: \qquad \frac{\partial T}{\partial y}=0 \qquad u=v=0 \tag{8}$$

The temperature T_0 and zero velocities have been used as initial conditions.

It can be shown that the solutions of Eqs. (1)–(8) are the same at inclination angle values equal to α and $(-\alpha)$. All we need to do is to replace boundary condition (5) at α by $T=T_0-T_1\sin\omega t$ at $(-\alpha)$. Therefore the heat transfer and flow characteristics have been studied for the range of inclination angle values from 0° to 90°.

Equations (1)–(8) have been considered in dimensionless form for convenience of numerical analysis. The characteristic parameters for this problem are

$$v^*=\frac{\mu}{\rho_0 L} \qquad \tau^*=\frac{\rho_0 L^2}{\mu} \qquad P^*=\rho_0 v^{*2} \tag{9}$$

If we define the parameter $f=\omega\rho_0 L^2/\mu$, then the argument in the dimensionless boundary condition will have the form $f\tau$.

For the dimensionless variables taken as

$$X = \frac{x}{L} \qquad Y = \frac{y}{L} \qquad \widetilde{P} = \frac{P + \rho_0 g(x \sin\alpha + y \cos\alpha)}{P^*}$$
$$\tau = \frac{t}{\tau^*} \qquad U = \frac{u}{v^*} \qquad V = \frac{v}{v^*} \qquad \theta = \frac{T - T_0}{T_1} \tag{10}$$

the system of equations (1)–(4) transforms to

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial \widetilde{P}}{\partial X} + \Delta U + \mathrm{Gr} \cdot \theta \sin \alpha \tag{11}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial \widetilde{P}}{\partial Y} + \Delta V + \mathrm{Gr} \cdot \theta \cos \alpha \tag{12}$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{13}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Pr}}\Delta\theta \tag{14}$$

and the dimensionless boundary conditions are

$$X = 0: \qquad U = V = 0 \qquad \theta = \sin f\tau \tag{15}$$

$$X = 1: \qquad U = V = 0 \qquad \theta = 0 \tag{16}$$

$$Y = 0: \qquad U = V = \frac{\partial \theta}{\partial Y} = 0 \tag{17}$$

$$Y = 1: \qquad U = V = \frac{\partial \theta}{\partial Y} = 0 \tag{18}$$

The dimensionless initial conditions are as follows:

$$\tau = 0 \qquad U = V = \theta = 0 \qquad \widetilde{P} = \mathrm{const} \tag{19}$$

Equations (11)–(19) have the following dimensionless parameters: Grashof number $\mathrm{Gr} = g\beta T_1 L^3 \rho_0^2/\mu^2$, Prandtl number $\mathrm{Pr} = c\mu/\lambda$, and boundary temperature oscillation frequency f.

PRELIMINARY DISCUSSION

Before we start numerical consideration, let us make do qualitative analysis of Eqs. (11)–(19). First, we will discuss the case of $\alpha = 0°$. The horizontal walls are adiabatic, and the period-averaged temperature of the left cavity side is equal to the temperature of the right wall. Therefore, during half of the oscillation period the cavity obtains the heat, and during the other half of the period it loses the same amount of the heat on the left wall. That is why the average heat flux through the enclosure remains equal to zero at $\alpha = 0°$.

Now, let us consider the case of $0^\circ < \alpha \leq 90^\circ$. When the temperature at $X = 0$ is greater than it is at $X = 1$, convective motion appears in the enclosure because of unstable stratification. During this part of the period the heat is transferred by both conduction and convection. When the temperature at $X = 0$ is lower than it is at $X = 1$, the stratification works against the convection and heat is transferred through the left wall mostly by conduction. Therefore the amount of heat transferring into the cavity through the wall with varying temperature could be greater than the heat losses at this wall. That means there should be nonzero period-averaged heat transfer through the enclosure.

It is clear that the depth of penetration and the amplitude of the temperature oscillations will depend on the dimensionless frequency f. It also can be noted that when $f \to \infty$ the penetration depth of oscillation will diminish to zero. Therefore there could be maxima of heat transfer and decrease of oscillation amplitude due to increasing frequency f. Additional eigenfrequencies will appear in this problem depending on Grashof number and inclination angle.

NUMERICAL DETAILS

The problem has been studied numerically by the control-volume method with the SIMPLER algorithm [4]. A grid of 50×50 control volumes has been used for the following values of parameters: $\mathrm{Pr} = 1$, $\mathrm{Gr} = 2 \times 10^5$, and $\mathrm{Gr} = 3 \times 10^5$. Results have been obtained for inclination angles $0 \leq \alpha \leq 90^\circ$ and various values of dimensionless frequency.

During the computations the time step used is $\Delta\tau = 4 \times 10^{-6}$. Thus, for instance, 25,000 time steps are needed to pass through one oscillation period at $f = 20\pi$.

Most of the results have been obtained at zero initial conditions. At least 10 oscillation periods needed to be calculated to get a solution that is independent on initial conditions.

RESULTS AND DISCUSSION

Let us consider the average heat flux through the wall at $X = 1$:

$$\mathrm{Nu}_r = -\int_0^1 \left.\frac{\partial\theta}{\partial X}\right|_{X=1} dY \tag{20}$$

Its variation in time at $\mathrm{Gr} = 2 \times 10^5$ and dimensionless frequency $f = 20\pi$ is shown in Figure 2 for the three values of inclination angle ($\alpha = 0^\circ$, $\alpha = 36^\circ$, and $\alpha = 89^\circ$). It can be seen that the behavior of this characteristic becomes more complex when the inclination angle is increasing. Thus, at $\alpha = 0^\circ$ the heat flux varies almost by sine law, but it has very complicated periodic dependence at $\alpha = 89^\circ$.

Figure 3 demonstrates similar results for the larger Grashof number, $\mathrm{Gr} = 3 \times 10^5$. Comparison of Figures 2 and 3 shows that increasing Gr leads to growth of the heat flux oscillation amplitude and to the complication of its behavior during the oscillation period.

The time dependence of Nu_r for a few values of dimensionless frequency at $\mathrm{Gr} = 3 \times 10^5$ and $\alpha = 60^\circ$ is shown in Figure 4. The amplitude of oscillation

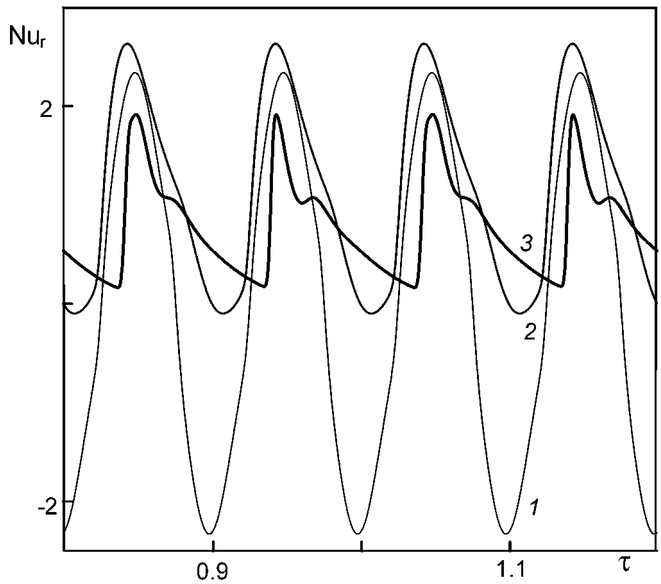

Figure 2. Heat flux Nu_r dependence on time for various values of α at $Gr = 2 \times 10^5$, $f = 20\pi$: 1, $\alpha = 0°$; 2, $\alpha = 36°$; 3, $\alpha = 89°$.

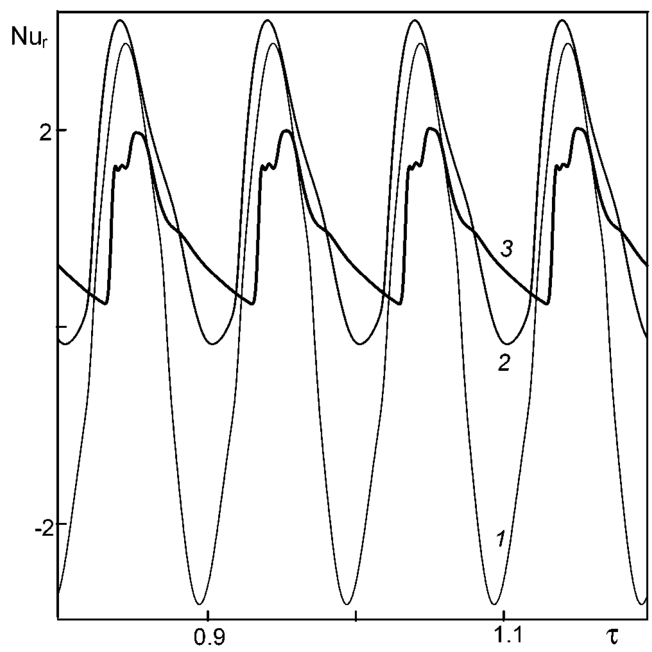

Figure 3. Heat flux Nu_r dependence on time for various values of α at $Gr = 3 \times 10^5$, $f = 20\pi$: 1, $\alpha = 0°$; 2, $\alpha = 36°$; 3, $\alpha = 89°$.

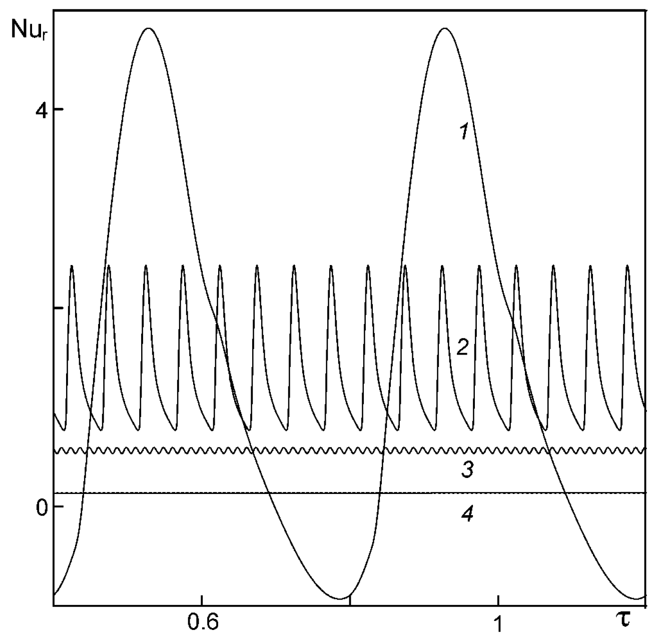

Figure 4. Heat flux Nu_r dependence on time at $Gr = 3 \times 10^5$ and $\alpha = 60°$ for various values of f: 1, $f = 5\pi$; 2, $f = 40\pi$; 3, $f = 160\pi$; 4, $f = 320\pi$.

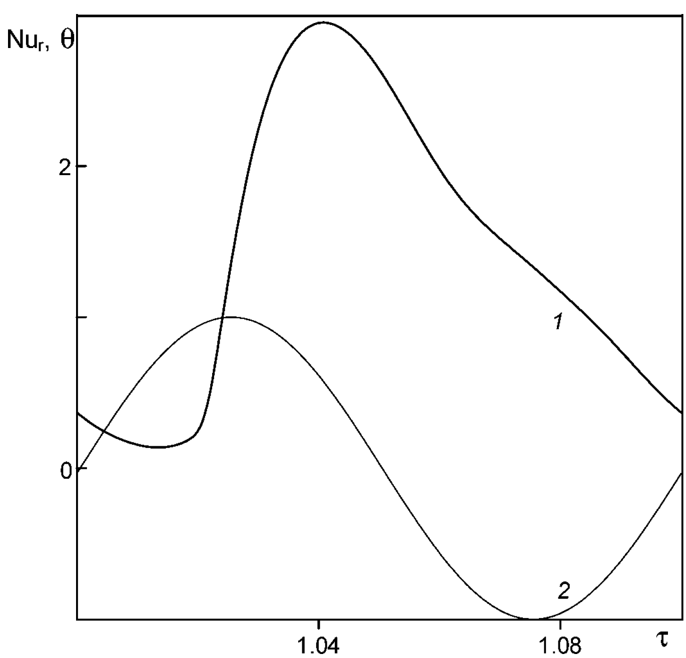

Figure 5. Variations of temperature at $X = 1$ and Nu_r in a period for $Gr = 3 \times 10^5$, $\alpha = 45°$, $f = 20\pi$: 1, Nu_r; 2, θ.

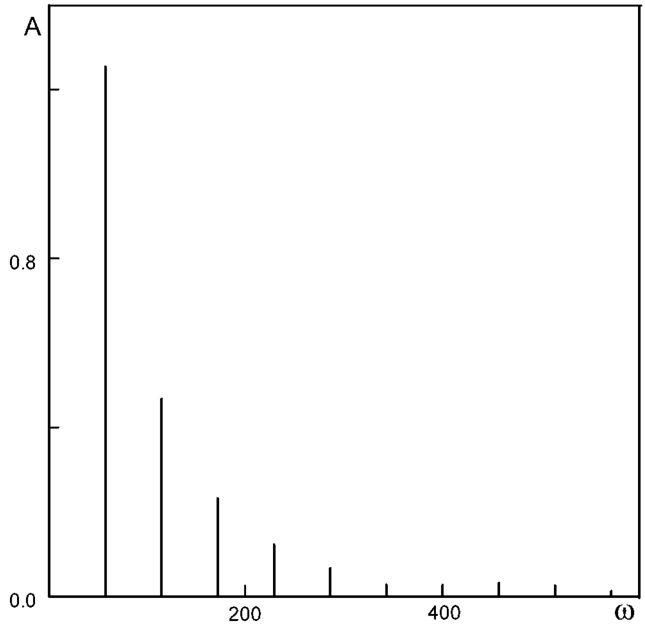

Figure 6. Eigenfrequencies of Nu_r oscillation at $Gr = 3 \times 10^5$, $\alpha = 45°$, $f = 20\pi$.

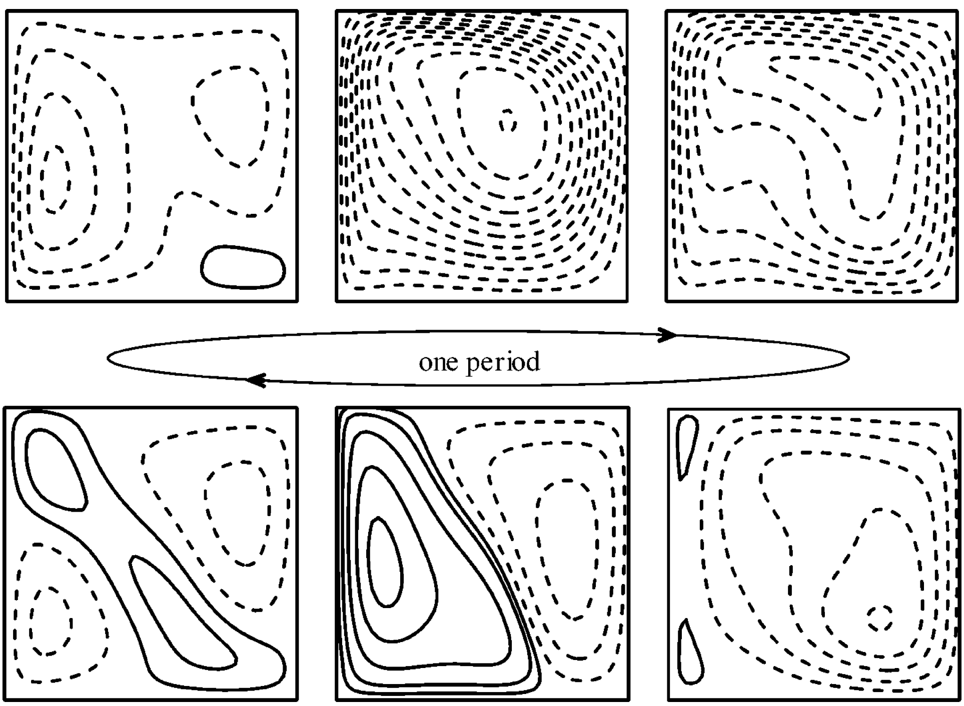

Figure 7. Representative plots of stream function (dashed lines correspond to negative values) during a period of oscillations. $Gr = 3 \times 10^5, f = 20\pi$, $\alpha = 45°$.

decreases while the frequency is growing, and the same behavior is known for the case of pure-conductivity problems. Figure 4 also shows that increasing frequency leads to decreasing heat transfer through the cavity, and that it will equal zero at $f \to \infty$.

Figure 5 shows the Nu_r variation during one period along with the temperature variation (which is described by nearly sine law) at $X = 1$ for fixed values of parameters $f = 20\pi$, $\alpha = 45°$, $\mathrm{Gr} = 3 \times 10^5$. The heat flux Nu_r is positive during all of the period, i.e., the heat is transferring in the direction that coincides with the direction of the X axis. It can also be seen that, together with the frequency f, the $\mathrm{Nu}_r(\tau)$ dependence has some additional eigenfrequencies. This fact is clearly depicted in Figure 6.

The flow transformations during the period are shown in Figures 7 and 8 by representative patterns of the stream function and temperature field. It can be seen that the flow structure is sufficiently changed in a period. There are one, two, and three cellular flows with different directions of rotation in the cavity.

Let us consider the time-averaged heat transfer through the cavity in a period:

$$\mathrm{Nu}_{\mathrm{av}} = -\frac{f}{2\pi}\int_0^{2\pi/f}\int_0^1 \left.\frac{\partial\theta}{\partial X}\right|_{X=1} dY\, d\tau \tag{21}$$

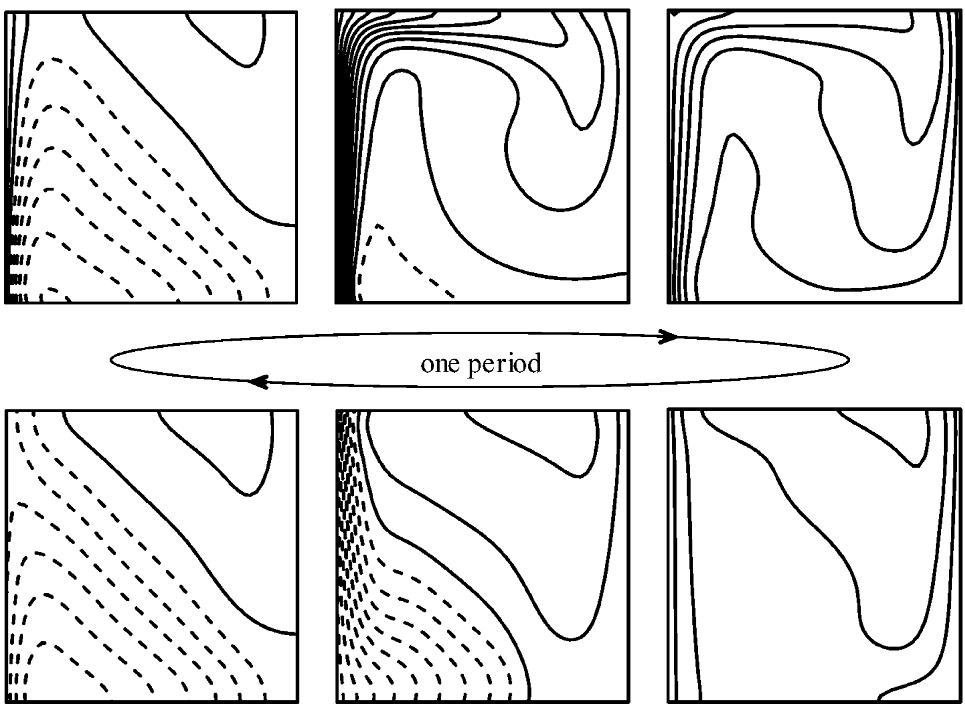

Figure 8. Representative plots of isotherms ($\Delta\theta = 0.1$, dashed lines correspond to negative values) during a period of oscillations. $\mathrm{Gr} = 3\times10^5$, $f = 20\pi$, $\alpha = 45°$.

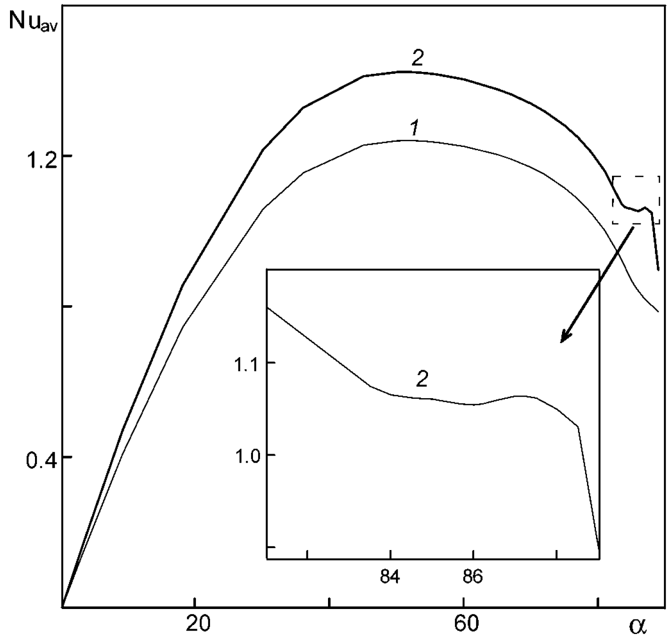

Figure 9. Time-averaged Nusselt numbers with inclination angle at Gr $= 2 \times 10^5$ (1) and Gr $= 3 \times 10^5$ (2) for $f = 20\pi$.

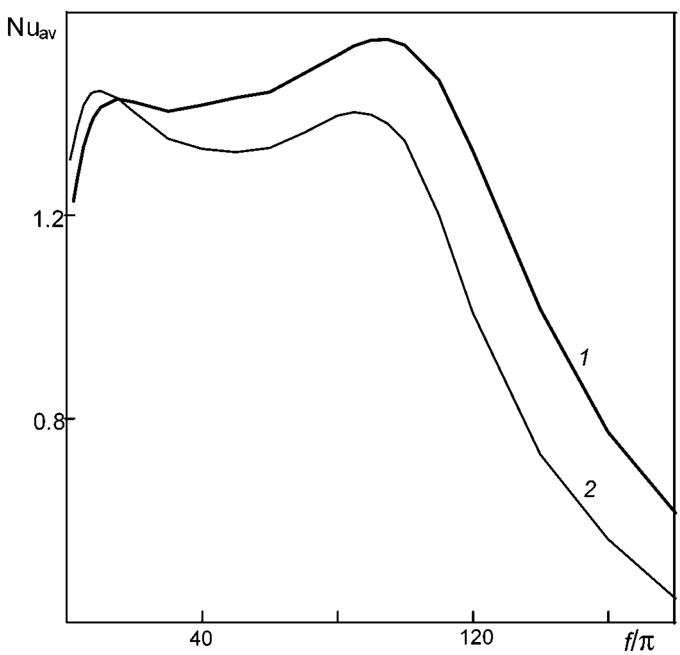

Figure 10. Time-averaged Nusselt numbers with frequency at $\alpha = 50°$ (1) and $\alpha = 60°$ (2) for Gr $=$ 3×10^5.

The dependence of Nu_{av} on inclination angle α is shown in Figure 9. It can be seen that there is zero heat flux through the cavity at $\alpha = 0°$ (as was expected). The heat transfer through the enclosure reaches its maximum at $\alpha = 54°$. Moreover, there are local maxima and minima near 90°; these are caused by flow transformations in the neighborhood of this angle, especially at $Gr = 3 \times 10^5$.

The dependencies of heat transfer on oscillation frequency for the fixed values of inclination angle ($\alpha = 50°$ and $\alpha = 60°$) are shown in Figure 10. The relation $Nu_{av}(f)$ is nonlinear and has rather complex behavior. For instance, the local minimum at $f = 50\pi$ and two local maxima at $f = 10\pi$ and $f = 85\pi$ are discovered for $\alpha = 60°$. And in the range of f from 5π to 100π the heat flux varies not more than 10%. The period-averaged heat transfer through the cavity diminishes very fast with increasing frequency at $f > 100\pi$.

CONCLUSIONS

The problem of natural-convective heat transfer through an inclined square enclosure with time-varied temperature on one of its wall has been studied numerically. It was essential here that the period-averaged temperature of one wall coincided with the constant temperature of the opposite wall.

It has been shown that at $\alpha \neq 0°$ there is a transfer of heat through the cavity in the direction of the X axis. The dependencies of the time-averaged heat flux on oscillation frequency have been obtained for fixed values of inclination angle with $Gr = 2\times10^5$ and $Gr = 3\times10^5$. The "heat transfer–inclination angle" relations also have been investigated for various values of temperature oscillation frequency.

It has been found that the maximal heat transfer through the studied enclosure with Grashof numbers equal to 2×10^5 and 3×10^5 can be achieved at $\alpha = 54°$ and $f = 20\pi$.

REFERENCES

1. M. Kazmierczak and Z. Chinoda, Buoyancy-Driven Flow in an Enclosure with Time Periodic Boundary Conditions, *Int. J. Heat Mass Transfer*, vol. 35, no. 6, pp. 1507–1518, 1992.
2. H. S. Kwak, K. Kuwahara, and J. M. Hyun, Resonant Enhancement of Natural Convection Heat Transfer in a Square Enclosure, *Int. J. Heat Mass Transfer*, vol. 41, pp. 2837–2846, 1998.
3. S. Y. Soong, P. Y. Tzeng, and C. D. Hsieh, Numerical Study of Bottom-Wall Temperature Modulation Effects on Thermal Instability and Oscillatory Cellular Convection in a Rectangular Enclosure, *Int. J. Heat Mass Transfer*, vol. 44, pp. 3855–3868, 2001.
4. S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, McGraw-Hill, New York, 1980.